

Does Positronium Form in the Universe ?

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Abstract. Positronium (the bound state of electron and positron) has been thought to be formed after proton decay ($> 10^{34}$ yr) through collisional recombination and then decays by pair annihilation, thereby changing the matter content of the universe. We revisit the issue of the formation of positronium in the long-term future of the universe in light of recent indication that the universe is dominated by dark energy and dark matter. We find that if the equation of state of dark energy w is less than $-1/3$ (including the cosmological constant $w = -1$), then the formation of positronium would not be possible, while it is possible through bound-bound transitions for $-1/3 \lesssim w \lesssim -0.2$, or through collisional recombination for $w \gtrsim -0.2$. The radiation from e^\pm pair annihilation cannot dominate over e^\pm , while that from proton decay will dominate over baryon and e^\pm for a while but not over dark matter.

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1. Introduction

Since the pioneering work by Dyson [1] (see also [2]), the physical processes that would occur in the far future of the universe have been studied by several authors [3, 4, 5, 6, 7]. One of interest then was the effect of proton decay on the matter content of the universe. It was found for an open universe that the decay of protons (with lifetime t_d) will produce nonthermal radiation which will dominate the universe from t_d to $10t_d$ and then matter domination follows [5]. Moreover, electrons and positrons from proton decay will bound to become positroniums and eventually annihilate [3]. However, the annihilation of positron-electron pair occurs so slowly that the radiation from pair annihilation cannot dominate the universe [4].

Today the situation in cosmology becomes much changed. The new ingredients after Page and McKee's paper are: (1) Cold dark matter; baryons are not dominant component of the universe, (2) Dark energy; the universe is presently dominated by energy density with negative pressure. As an additional minor point, we note that the lower-bound of the proton decay, $t_d > 10^{33}\text{yr}$ [8], is much larger than the previous bound ($t_d > 10^{30}\text{yr}$). The purpose of this paper is to revisit the nature of the long-term future of the universe taking into account these new ingredients after Page and McKee's study [4]. Throughout this paper, we assume that the laws of physics continue to hold unchanged. In particular, we assume that the charge of electron does not vary with time.

2. Positronium Formation in an Expanding Universe

According to the grand unified theory (GUT) of elementary forces, all baryons are unstable. The life time of a baryon t_d is predicted to be typically some 10^{34} years for (supersymmetric) GUT models. The positrons and electrons, when first produced from nucleon decay, will have energies of several hundred MeV so that they will appear as relativistic particles until the universe has expanded by a factor of the decay energy divided by the electron mass. Let q_- be the electron mass divided by the average decay energy per the electron mass and q_+ be the same ratio for positrons. Let f_- be the average number of electrons produced per nucleon decay and f_+ be the same quantity for positrons. For SU(5) GUT model,[‡] assuming that 16% of the nucleons in the universe are neutrons, they are given by [5, 9]

$$q_- = 5.9 \times 10^{-3}, \quad f_- = 0.54 \tag{1}$$

$$q_+ = 2.1 \times 10^{-3}, \quad f_+ = 1.38. \tag{2}$$

[‡] We adopt non-supersymmetric SU(5) GUT as an example only for simplicity. Our qualitative results will not be sensitive to the detail of the model.

Thus, baryon density ρ_B , e^\pm 's density ρ_e and radiation density ρ_r evolve according to §[5]

$$\dot{\rho}_B = -3H\rho_B - t_d^{-1}\rho_B \quad (3)$$

$$\dot{\rho}_e = -3H\rho_e + \frac{m\rho_B}{m_p t_d} (q_- f_- \theta(t_- - t_0) + q_+ f_+ \theta(t_+ - t_0)) \quad (4)$$

$$\dot{\rho}_r = -4H\rho_r + t_d^{-1}\rho_B - \frac{m\rho_B}{m_p t_d} (q_- f_- \theta(t_- - t_0) + q_+ f_+ \theta(t_+ - t_0)), \quad (5)$$

$$H^2 = \frac{8\pi G}{3}(\rho_{DE} + \rho_{DM} + \rho_B + \rho_e; \rho_r) \quad (6)$$

where m is the mass of electron(positron), m_p is the proton mass. $t_-(t_+)$ is the time when the universe expand $1/q_-(1/q_+)$ times from now (t_0), and ρ_{DE} (ρ_{DM}) is the energy density of dark energy (matter). The second term in Eq.(4) includes the effect that electrons (and positrons) from proton decay are initially considered radiation and will become matter when the universe is $1/q_-$ ($1/q_+$) times expanded. Assuming that the universe is dominated by dark energy with the equation-of-state w and dark matter with $\rho_{DM}(t_0) = 6 \times \rho_B(t_0)$ which is stable^{||} and taking $\rho_r = 0$ and $\rho_e = (m/m_p)\rho_B(t_0)(f_+ - f_-)$ (from charge neutrality) at the present time $t = t_0$, we solve the above equations. The results are shown in Fig. 1 for $w = -0.3$ and $w = -1$. We find that radiation can dominate over baryon, electron and positron for only a short period after proton decay for $w = -0.3$.

2.1. Necessary Condition

When a baryon with mass m_p decays, a significant fraction of its energy will go into e^\pm s (their mass m) with average rms momentum and energy $p = \gamma m$ at that time t_d , where γm is roughly the fraction of energy m_p of each decay that goes into each e^\pm produced, say $\gamma \sim m_p/2m \sim 10^3$ in accord with the previous example ($1/q_-, 1/q_+$). As the universe expands, in the absence of non-gravitational interactions the rms momentum will redshift to $\gamma m(a_d/a)$. Once this becomes less than m (when $a > \gamma a_d$), the average kinetic energy per e^\pm will be $p^2/2m \sim \gamma^2 m(a_d/a)^2$. In the following, we shall consider a cosmological model with dark energy with equation-of-state w . Hence the expansion law is $a \propto t^{2/3(1+w)}$.

As Barrow and Tipler argued, a positronium will form if the average energy of an electron E at average distance r from a positron

$$E \sim \gamma^2 m(a_d/a)^2 - e^2/r \quad (7)$$

§ Here we have assumed that all of the baryons in the universe are free at the epoch of proton decay. In reality, however, a large fraction of the baryons would be locked up in degenerate remnants (mostly in white dwarfs) at this future time. When protons decay within dwarfs, the positrons annihilate with electrons, and the resulting high energy photons are reprocessed into low energy photons. As a result, protons that are locked up in degenerate remnants are essentially lost to the processes discussed here. The subsequent analyses would apply to the baryons that will not be processed into stars and stellar remnants.[10, 7]

|| We assume that dark matter is stable. The subsequent discussion will be greatly changed for unstable dark matter, which will be strongly model-dependent however.

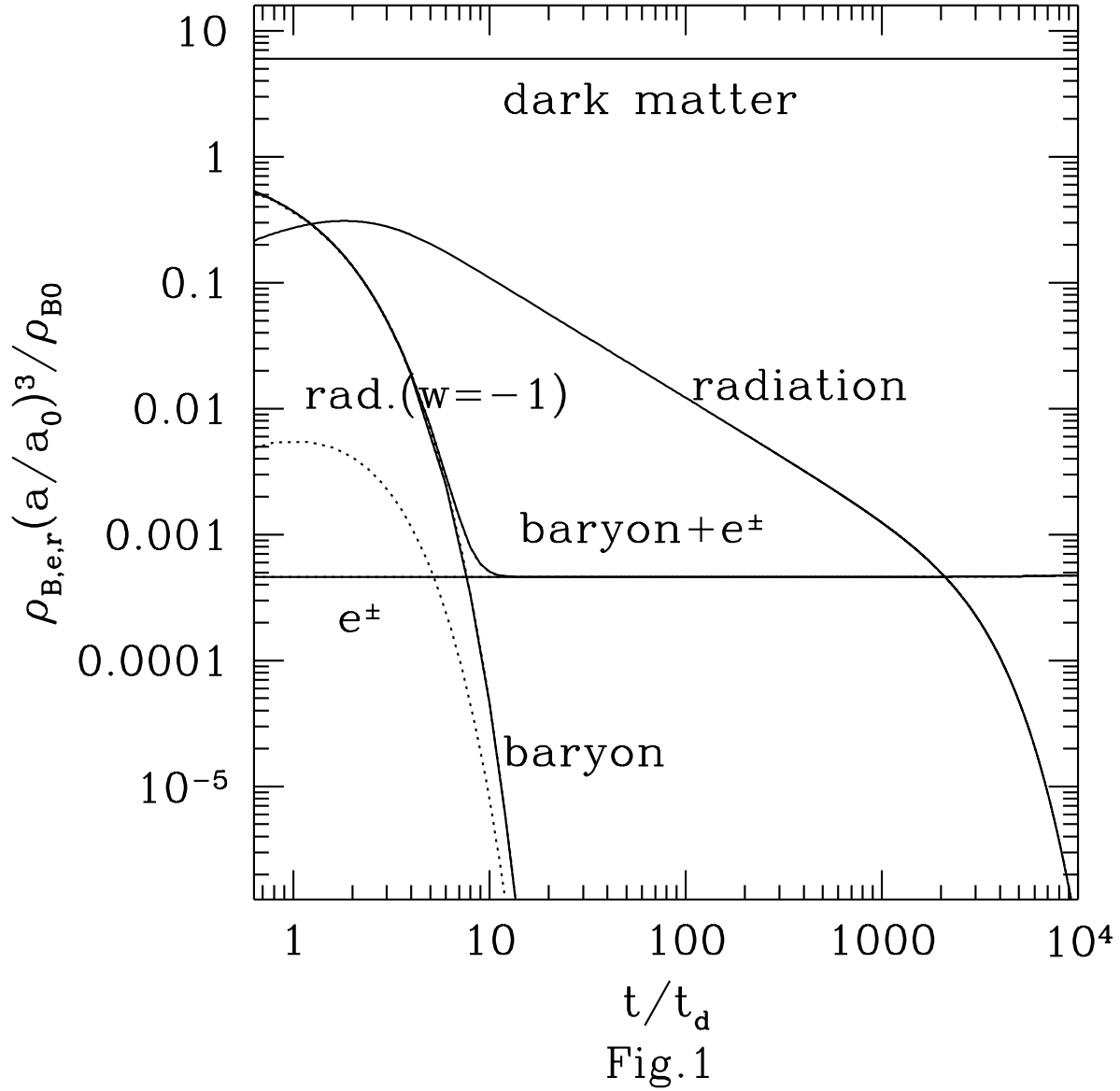


Figure 1. Evolution of ρ_B , ρ_e and ρ_r for $w = -0.3$ (solid lines) and for $w = -1$ (dashed lines). a_0 corresponds to the scale factor at the present time t_0 . ρ_e does slightly increase after baryon decay, although it is barely seen in the figure.

is negative [3].¶ Note that the Coulomb energy decays slower than the kinetic energy and hence E will be eventually negative. Here the mean separation r is estimated from the number density of e^\pm , N

$$r \sim N^{-1/3}. \quad (8)$$

N is related to the number density of baryon at the decay time N_d as

$$N \sim N_d (a_d/a)^3. \quad (9)$$

¶ This is the case for bound-bound transition. Positronium can be formed much earlier through collisional recombination [4]. The detailed analysis is given in Appendix.

N_d is written in terms of the ratio of baryon number density to entropy density $n_B/s \sim 10^{-10}(\Omega_B h^2/10^{-2})$ as

$$N_d \sim n_{B0}(a_0/a_d)^3 \sim 10^{-7} (n_{B0}^{-1}/10^{-10}) (a_0/a_d)^3 \text{cm}^{-3}, \quad (10)$$

where a_0 is the scale factor at present. An electron-positron pair will bound if $E < 0$ when $a > a_*$, where from Eq. (7)

$$a_*/a_d \sim \gamma^2 m e^{-2} N_d^{-1/3} \sim 10^{21} 10^{16/(1+w)} (t_d/10^{34} \text{yr})^{2/3(1+w)}. \quad (11)$$

In an accerelating universe ($w < -1/3$), another condition for the formation of positronium arises: repulsive gravitational force sourced by dark energy should not overcome the attractive Coulomb force:

$$e^2/r^2 > -mr\ddot{a}/a = -mr(1+3w)H^2/6. \quad (12)$$

The question is whether Eq.(12) is satisfied at that time. From

$$r \sim e^2 \gamma^{-2} m^{-1} (a_*/a_d)^2 \quad (13)$$

and

$$H^{-1} \sim H_0^{-1} (a/a_0)^{3(1+w)/2} \sim 10^{34} (t_d/10^{34} \text{yr}) (a/a_d)^{3(1+w)/2} \text{yr}, \quad (14)$$

it is required that (for $w < -1/3$)

$$a_*/a_d < 10^{49/(1-w)} (t_d/10^{34} \text{yr})^{2/3(1-w)}, \quad (15)$$

where we have neglected an unimportant numerical coefficient coming from $|1+3w|$ in Eq.(12). Two conditions Eq.(11) and Eq.(15) are compatible if (for $w < -1/3$)

$$10^{21} 10^{16/(1+w)} (t_d/10^{34} \text{yr})^{2/3(1+w)} < 10^{49/(1-w)} (t_d/10^{34} \text{yr})^{2/3(1-w)}. \quad (16)$$

Taking the logarithm, the above condition is rewritten as

$$\frac{4w}{3(1-w^2)} \log(t_d/10^{34} \text{yr}) + \frac{33+65w}{1-w^2} - 21 > 0. \quad (17)$$

It is easily found that the left-hand-side of Eq.(17) is an increasing function of w (for $-1 < w < -1/3$) and negative as long as

$$w < -1/3 \quad (18)$$

for $t_d > 10^{33} \text{yr}^+$ Hence positronium will not form in the universe dominated by dark energy with $w < -1/3$.^{*} We will restrict ourselves to the case with $w > -1/3$ hereafter.

The formation of positronium through collisional recombination is studied in appendix. Here we only give the results: the typical time scale for the formation is $\sim 10^{83} \text{yr} (t_d/10^{34} \text{yr})$ and the size of positronium is $\sim 10^{45} \text{Mpc}$ for $w = -0.1$ (see Fig.3 and Fig.4).

⁺ The condition would apply to the equation of state at the time of formation. For dark energy which has the present equation of state $w < -1/3$ temporarily and then would increase toward $w > -1/3$ (for example, quintessence axion [11]), the formation of positronium would be possible.

^{*} This does not mean that positronium will never form in the universe with $w < -1/3$. Although some small bit of activity always takes place, it will be exponentially suppressed.[12]

3. Positronium Decay and Annihilation

We have seen that positronium will be formed if $w > -1/3$. Positronium, once formed long after nucleon decay, will decay toward ground state and then pair-annihilate. One can use the corresponding principle to calculate the time within which the positronium will spiral into the ground state by dipole radiation (for large n) [15, 4]

$$t_{spiral} \sim m^{-1} e^{-10} n^6. \quad (19)$$

From Eq.(62) in Appendix up to n_{max} , the fraction of free e^\pm 's evolves as

$$y_e \propto a^{-3(2-\alpha)/5\alpha}, \quad (20)$$

where the expansion law is $a \propto t^\alpha$, $\alpha \equiv 2/3(1+w)$. Then a lifetime of decay to the ground state is

$$t_{spiral} \propto n_{max}^6 \propto y_e^{-1} a^3 \propto a^{6(1+2\alpha)/5\alpha}. \quad (21)$$

Expressing y_e in terms of t_{spiral} gives the logarithmic annihilation rate coefficient [4]

$$\frac{\beta}{t} = \frac{2-\alpha}{2+4\alpha} \frac{1}{t} = \frac{2+3w}{7+3w} \frac{1}{t}. \quad (22)$$

Thus, after the formation of positronium, e^\pm 's density ρ_e and radiation density ρ_r evolve according to

$$\dot{\rho}_e = -3H\rho_e - \beta t^{-1}\rho_e \quad (23)$$

$$\dot{\rho}_r = -4H\rho_r + \beta t^{-1}\rho_e. \quad (24)$$

$$(25)$$

Assuming that $\rho_e = \rho_{ed}$ and $\rho_r = \rho_{rd}$ at $t = t_d$, the solutions are

$$\rho_e = \rho_{ed}(t/t_d)^{-(3\alpha+\beta)} \quad (26)$$

$$\rho_r = \frac{\beta\rho_{ed}}{\alpha-\beta} \left((t/t_d)^{-(3\alpha+\beta)} - (t/t_d)^{-4\alpha} \right) + \rho_{rd}(t/t_d)^{-4\alpha}. \quad (27)$$

The asymptotic ratio of ρ_e to ρ_r is given by

$$\frac{\rho_e}{\rho_r} = \frac{\alpha-\beta}{\beta} = \frac{8-9w-9w^2}{3(2+3w)(1+w)}. \quad (28)$$

In Fig. 2, the ratio is shown as a function of w . The ratio is greater than $4/3$ for $w < 0$. Hence, the radiation by e^\pm pair annihilation cannot dominate over e^\pm . This is because in the universe with dark energy the expansion rate is higher and radiation suffers from much larger redshift.

4. Summary

We have investigated the matter content in the long-term future of the universe. After the proton decay, radiation becomes dominant over baryon and e^\pm for a while and then is followed by them again but it will not dominate over dark matter.

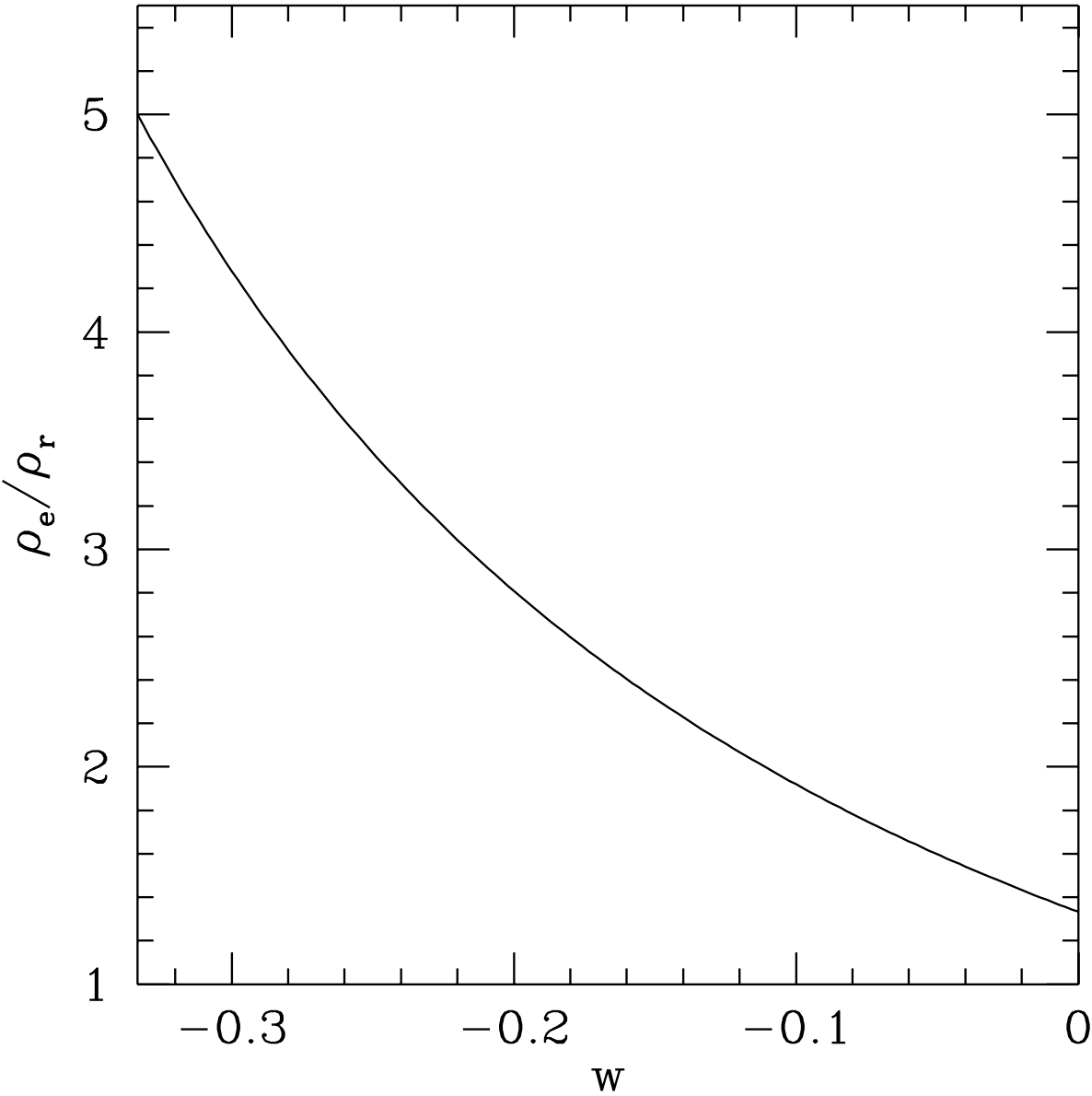


Fig.2

Figure 2. The ratio of e^\pm 's density to radiation density.

now	Dark Energy	Dark Matter	Baryon	Radiation	e^\pm
10^{34} yr	Dark Energy	Dark Matter	Radiation	Baryon	e^\pm
10^{38} yr	Dark Energy	Dark Matter	e^\pm	Radiation	
10^{83} yr	Dark Energy	Dark Matter	e^\pm /Ps	Radiation	

Table 1. The energy content of the universe assuming $t_d = 10^{34}$ years. PS denotes positronium.

We have also investigated the possibility of formation of positronium after proton decay. We have found that positronium will not be formed in the universe if the equation of state of dark energy w is less than $-1/3$. Positronium will be formed after 10^{83} years if the equation of state of dark energy is $w > -1/3$, and radiation will be produced by positronium decay. However, it will never dominate over matter due to the slow annihilation rate. The results are summerized in Table 1.

Acknowledgments

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Appendix: Collisional Recombination

In this appendix, following Page and McKee [4], we give the detailed analysis of positronium formation by collisional recombination.

As shown by Page and McKee [4], radiative processes are unimportant for free e^\pm . Therefore positronium will form by collisional recombination,

$$e^- + e^+ + e^\pm \rightarrow \text{Ps}_n + e^\pm, \quad (29)$$

where Ps_n denotes positronium with principal quantum number n . Once formed, positronium will tend to decay toward the ground state or at least an S state and then annihilate. At first, the transitions will be induced mainly by collisions (for large n states),

$$\text{Ps}_n + e^\pm \rightarrow \text{Ps}_{n'} + e^\pm \quad (30)$$

$$\text{Ps}_n + \text{Ps}_{n''} \rightarrow \text{Ps}_{n'} + \text{Ps}_{n'''}. \quad (31)$$

Let N_e and N_n be the number densities of free e^\pm 's and Ps_n 's. We introduce the fractions of free e^\pm 's and Ps_n 's

$$y_e = N_e/N, \quad y_n = N_n/N, \quad (32)$$

where N is the total number density of all e^\pm 's.

The transitions Eq.(29) and Eq.(30) and their inverses occur at rates proportional to the densities of the participating particles, The rate is given by

$$\frac{dy_n}{dt} = \frac{1}{4}N^2y_e^3C_{i,n} - Ny_eC_{n,i}y_n + Ny_e \sum_{n'} (\alpha(n', n)y_{n'} - \alpha(n, n')y_n), \quad (33)$$

where $C_{i,n}$ is the collisional recombination rate coefficient to the n th level, $C_{n,i}$ is the collisional ionization rate coefficient from the n th level, and $\alpha(n, n')$ is the collisional transition rate coefficient. They are defined by

$$C_{n,i} = \langle \sigma^{ion}(n)v \rangle \quad (34)$$

$$\alpha(n, n') = \langle \sigma(n, n')v \rangle, \quad (35)$$

where $\sigma^{ion}(n)$ is the cross section for ionization from the n th level, $\sigma(n, n')$ is the cross section for transitions from the n th to the n' th level, and the averages are taken over the Maxwellian distribution of the relative velocities. By detailed balance $C_{i,n}$ is given by

$$C_{i,n} = 8n^2(\pi/mT)^{3/2} \exp(me^4/4n^2T) C_{n,i}, \quad (36)$$

where T is the e^\pm temperature, $T \sim \gamma^2 m(a_d/a)^2$. Note that the binding energy of positronium is $me^4/4n^2T$.

The ionization cross section of an atom is given in [13]

$$\sigma^{ion} = \frac{\pi e^4(5E + 2I_n)(E - I_n)}{3E^2 I_n(E + 3I_n)}, \quad (37)$$

where E is the energy of the incident electron and I_n is the ionization energy of the atom ($I_n = me^4/4n^2T$ for positronium). However, for our crude purposes, a rough expression at high and low energies is adequate:

$$\sigma^{ion} \sim \frac{e^4(E - I_n)}{E^2 I_n}. \quad (38)$$

In terms of the ratio of ionization energy to temperature $x_n = I_n/T$, $C_{n,i}$ and $C_{i,n}$ are given by using Eq.(34) and Eq.(36)

$$C_{n,i} \sim n^2 m^{-3/2} T^{-1/2} (1 + x_n)^{-1} \exp(-x_n), \quad (39)$$

$$C_{i,n} \sim n^4 m^{-3} T^{-2} (1 + x_n)^{-1}. \quad (40)$$

The collisional cross sections for transitions between highly excited levels of hydrogen have been considered in several papers [13, 14] (see [4] for a list of references). An interpolation formula in [14]) gives the cross section which is accurate to 15% for $I_n \lesssim E \ll m, e^2 n^{-1} \lesssim v \ll 1$. If we drop numerical factors and logarithms, then for $1 \ll s \equiv n' - n \ll n$ the cross section become

$$\sigma(n, n + s) \sim n^4 s^{-3} m^{-1} E^{-1}, \quad me^4 n^{-1} s^{-1} < E < m \quad (41)$$

$$\sigma(n, n + s) \sim n^6 s^{-1} e^{-8} m^{-3} E, \quad me^4 n^{-2} < E < me^4 n^{-1} s^{-1}. \quad (42)$$

From Eq.(35), we obtain

$$\alpha(n, n + s) \sim n^4 s^{-3} m^{-3/2} T^{-1/2}, \quad me^4 n^{-1} s^{-1} < E < m \quad (43)$$

$$\alpha(n, n + s) \sim n^6 s^{-1} e^{-8} m^{-7/2} T^{3/2}, \quad me^4 n^{-2} < E < me^4 n^{-1} s^{-1}. \quad (44)$$

By detailed balance, the deexcitation rate coefficients are give by

$$\alpha(n + s, n) = (1 + s/n)^{-2} \exp(x_n - x_{n+s}) \alpha(n, n + s). \quad (45)$$

It is apparent that from Eq.(43) and Eq.(44) that for $T > me^4 n^{-2}$ the dominant bound-bound transitions are for relatively small changes s in the level n . Hence if one views

$$\alpha_s = \alpha_s(n, t) \equiv \alpha(n, n + s) \quad (46)$$

$$y = y(n, t) \equiv y_n \quad (47)$$

as smooth functions of n as well as of t , uses the detailed balanced relation Eq.(45), and expands Eq.(33) to second order in $s = n' - n$, one gets a diffusion equation

$$\begin{aligned} \frac{\partial y}{\partial t} \simeq & \frac{1}{4} N^2 y_e^3 C_{i,n} - N y_e C_{n,i} y \\ & + \frac{\partial}{\partial n} \left(N y_e \sum_{s=1}^n \alpha_s s^2 n^2 \exp(x_n) \frac{\partial}{\partial n} (\exp(-x_n) y n^{-2}) \right). \end{aligned} \quad (48)$$

Neglecting a $\ln n$ factor, the sum over s gives

$$\sum_{s=1}^n \sim n^4 m^{-3/2} T^{-1/2}, \quad m e^4 n^{-2} < T < m. \quad (49)$$

Now if we define the fraction of e^\pm 's in each individual positronium state as

$$z \equiv n^{-2} y(n, t) / 2, \quad (50)$$

then for nonrelativistic temperatures well above the ionization temperature ($T \ll m$ and $x_n \ll 1$),

$$\frac{\partial z}{\partial t} \sim \frac{N y_e}{m^{3/2} T^{1/2}} \left[n^2 \left(\frac{N y_e^2}{m^{3/2} T^{3/2}} - z \right) + \frac{1}{n^2} \frac{\partial}{\partial n} \left(n^6 \frac{\partial z}{\partial n} \right) \right]. \quad (51)$$

When $y_e \sim 1$, the population of positronium states with $x_n \ll 1$ increases toward the limiting value

$$z_S(n, t) \sim N y_e^2 (m T)^{-3/2} \quad (52)$$

$$\sim N_d y_e^2 (\gamma m)^{-3} \sim 10^{-48} 10^{-48/(1+w)} y_e^2 (t_d / 10^{34} \text{yr})^{-2/(1+w)}, \quad (53)$$

which is the Saha equilibrium value for $x_n \ll 1$. The population reaches this nearly stationary equilibrium only for levels in which the diffusion time is short compared with the expansion time t . Populations are only filled up to z_S for

$$n > n_{eq}(t) \sim y_e^{-1/2} N^{-1/2} (m T)^{3/4} t^{-1/2} \quad (54)$$

$$\sim 10^{-10} 10^{24/(1+w)} y_e^{-1/2} (t_d / 10^{34} \text{yr})^{(1-w)/(2+2w)} (t/t_d)^{(1-3w)/(6+6w)} \quad (55)$$

Since positronium must have a orbital size $r_n (= 2m^{-1} e^{-2} n^2)$ less than the mean separation $(y_e N)^{-1/3}$ between free e^\pm 's,

$$n < n_{max}(t) \sim m^{1/2} e (y_e N)^{-1/6} \quad (56)$$

$$\sim 10^6 10^{8/(1+w)} y_e^{-1/6} (t_d / 10^{34} \text{yr})^{1/3(1+w)} (t/t_d)^{1/3(1+w)}. \quad (57)$$

As the universe expands, $n_{max}(t)$ increases and more Ps_n levels become available to be filled up to population z_S . The filling up of levels with $x_n < 1$ decreases the fraction of free e^\pm 's by

$$dy_e \sim -z_S n_{max}^2 dn_{max} \sim -y_e^{3/2} a^{1/2} da / a_2^{3/2}, \quad (58)$$

where the time scale a_2 is

$$a_2 / a_d \sim \gamma^2 m e^{-2} N_d^{-1/3} \sim 10^{21} 10^{16/(1+w)} (t_d / 10^{34} \text{yr})^{2/3(1+w)}. \quad (59)$$

This is the same as a_* given by Eq.(11) since the binding energy, I_n , is smaller than the average kinetic energy.

However, for level $x_n > 1$ or

$$n < n_{\text{thermal}} = (me^4/4T)^{1/2} \quad (60)$$

$$\sim e^2 \gamma^{-1} (a/a_d) \sim 10^{-5} (t/t_d)^{2/3(1+w)}, \quad (61)$$

the exponential increase in the Saha equilibrium value above z_S (for small x_n) allows the value of z to exceed z_S if the collisional recombination rates are fast enough. Bound-bound transitions are less important in this case, and ionization becomes exponentially damped for $x_n > 1$, so

$$\frac{\partial z}{\partial t} \simeq \frac{N^2 y_e^3}{8n^2} C_{i,n} \sim \frac{n^4 N^2 y_e^3}{m^4 e^4 T} \sim \frac{n^4 N_d^2 y_e^3}{\gamma^2 m^5 e^4} (a/a_d)^{-4}. \quad (62)$$

This implies that once the ionization becomes unimportant for levels n , that level will fill up to population

$$z \sim n^4 N_d^2 y_e^3 \gamma^{-2} m^{-5} e^{-4} (a/a_d)^{-4} t. \quad (63)$$

This in turn depletes the free e^\pm 's by

$$dy_e \sim -z n_{\text{thermal}}^2 dn_{\text{thermal}} \sim y_e^3 a^{2+3(1+w)/2} da/a_1^{3+3(1+w)/2}, \quad (64)$$

with the time scale a_1 , where

$$a_1/a_d \sim [e^{-10} \gamma^9 m^5 N_d^{-2} H_0 (a_0/a_d)^{3(1+w)/2}]^{2/(9+3w)} \quad (65)$$

$$\sim 10^{(296+104w)/(9+3w)/(1+w)} (t_d/10^{34} \text{yr})^{2(3-w)/(9+3w)/(1+w)}. \quad (66)$$

In Fig. 3, t_1 and t_2 are plotted as a function of w , where $t_1(t_2)$ is the time corresponding to $a_1(a_2)$. Typically $t_1 \sim 10^{83} \text{yr} (t_d/10^{34} \text{yr})$.

For $-0.2 \lesssim w < 0$, $t_1 < t_2$, Most e^\pm 's will bind around the time t_1 , going into Ps_n levels n somewhat below

$$n_1 = n_{\text{thermal}}(t_1) \sim 10^{-5} (a_1/a_d). \quad (67)$$

These positronium states are very loosely bound, with radii

$$r_n \sim n_1^2 e^{-2} m^{-1} \quad (68)$$

$$\sim 10^{-43} 10^{(592+208w)/(9+3w)/(1+w)} (t_d/10^{34} \text{yr})^{4(3-w)/(9+3w)/(1+w)} \text{Mpc}. \quad (69)$$

$r_n \sim 10^{30} \text{Mpc}$ for $w = -0.1$.

For $w \lesssim -0.2$, however, since the expansion rate of the universe becomes higher, collisional recombination is no longer effective, and hence $t_1 > t_2$. The typical size of positronium is

$$r \sim e^2 \gamma^{-2} m^{-1} (a_2/a_d)^2 \quad (70)$$

$$\sim 10^{-1} 10^{32/(1+w)} (t_d/10^{34} \text{yr})^{4/3(1+w)} \text{Mpc}. \quad (71)$$

$r \sim 10^{45} \text{Mpc}$ for $w = -0.3$. The results are shown in Fig.4.

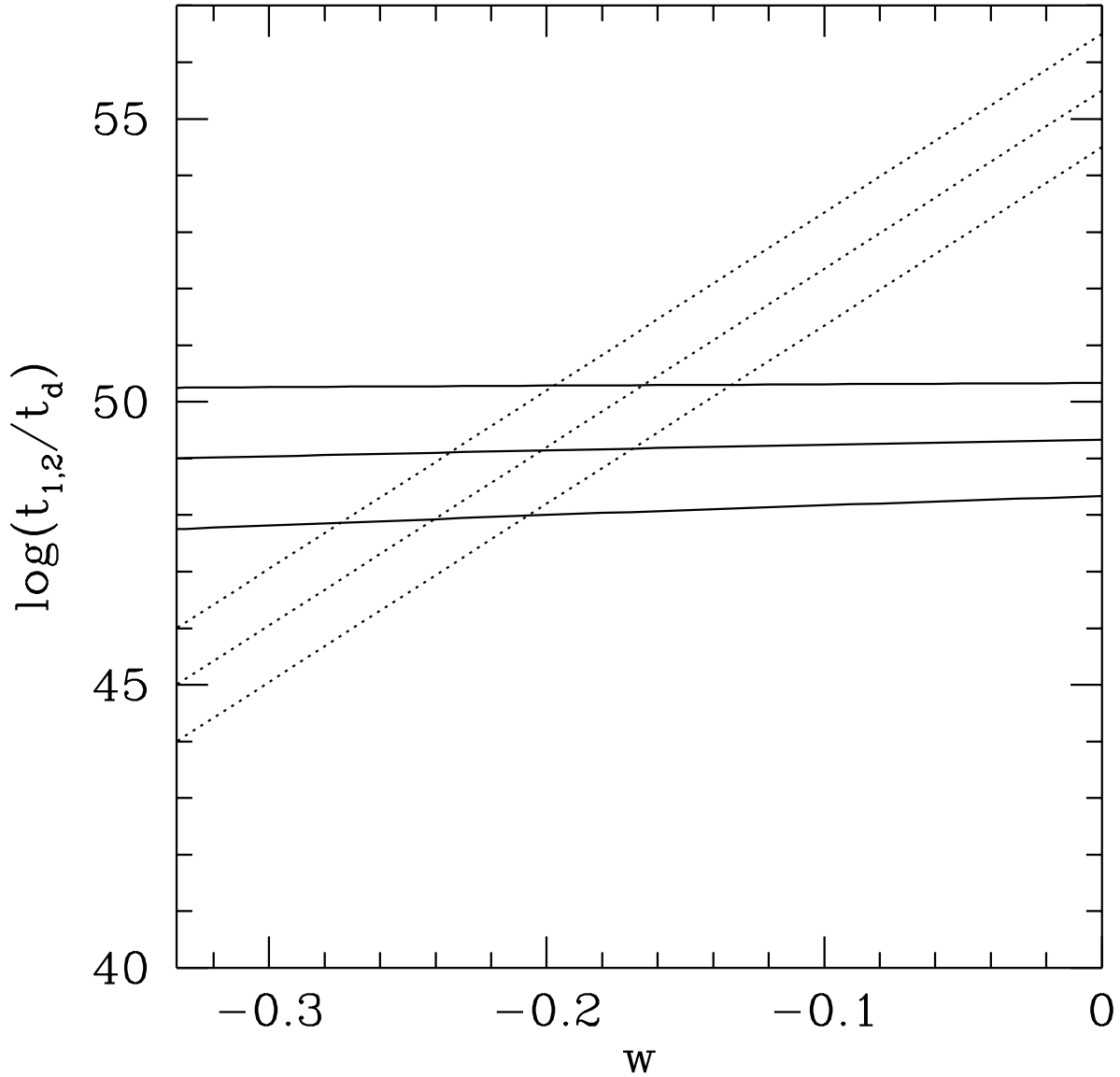


Fig.3

Figure 3. The formation times by collisional recombination (t_1 : solid lines) and by bound-bound transitions (t_2 : dashed lines) as functions of w . Three line are for $t_a = 10^{33}\text{yr}, 10^{34}\text{yr}, 10^{35}\text{yr}$ from top to bottom.

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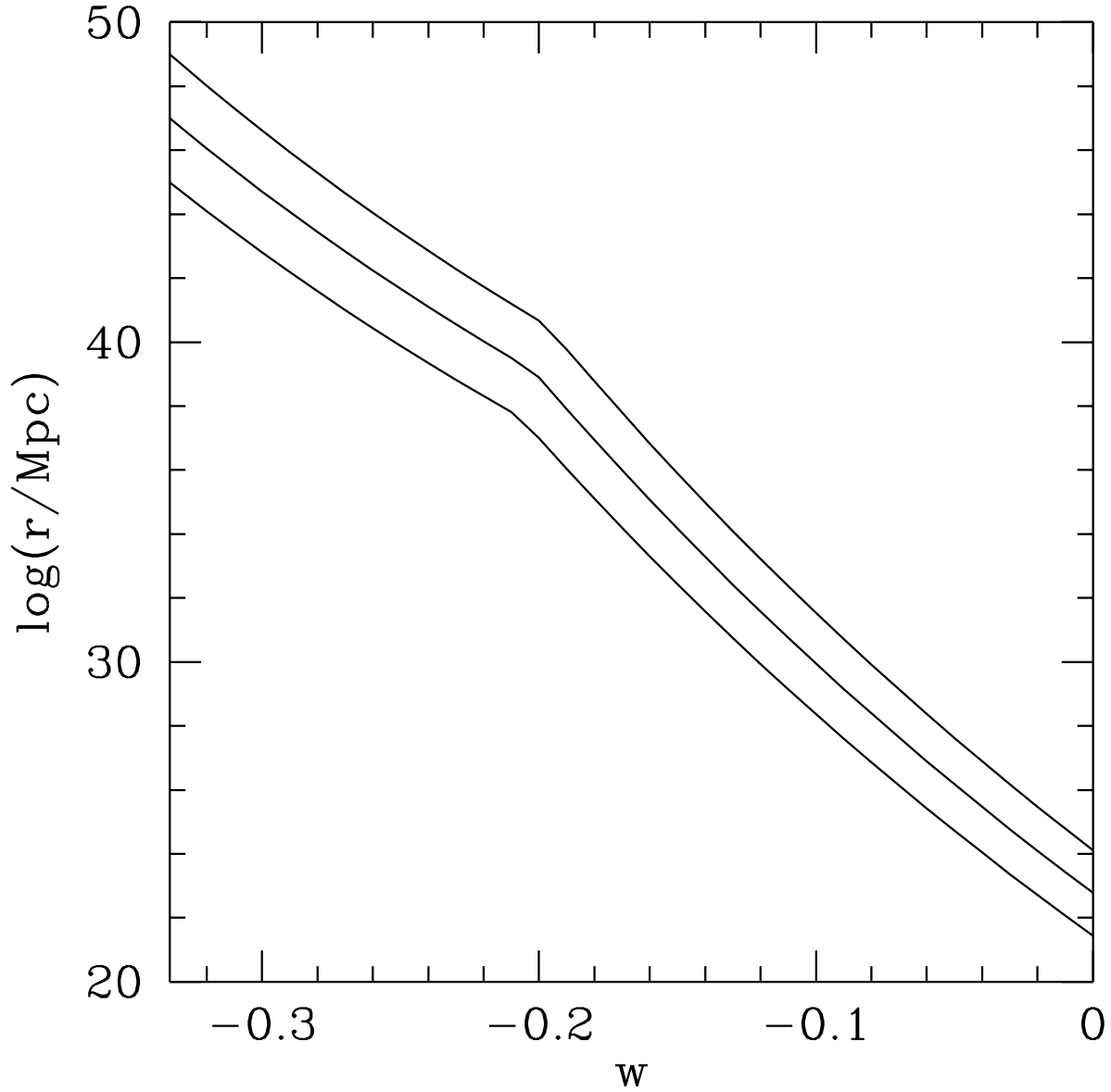


Fig.4

Figure 4. The size of positronium as a function of w . Three line are for $t_a = 10^{33}\text{yr}, 10^{34}\text{yr}, 10^{35}\text{yr}$ from bottom to top.

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